

Lesson Objectives

- Solve real-world problems involving area and circumference of circles.
- Solve real-world problems involving semicircles, quadrants, and composite figures.

garn Use the formula for circumference to solve real-world problems.

a) A circular mat has a diameter of 53 centimeters. Lily wants to sew a decorative braid around the mat. How many centimeters of braid does she need? Give your answer to the nearest tenth of a centimeter. Use 3.14 as an approximation of π .

Circumference of mat = πd $\approx 3.14 \cdot 53$ = 166.42 cm Write formula. Substitute. Multiply. Round to the nearest tenth of a centimeter.

Lily needs approximately 166.4 centimeters of decorative braid.

≈ 166.4 cm

b) A metalworker cuts out a large semicircle with a diameter of 28 centimeters. Then the metalworker cuts a smaller semicircle out of the larger one and removes it. The diameter of the semicircular piece that is removed is 14 centimeters. Find the distance around the shape after the smaller semicircle is removed. Use ²²/₇ as an approximation for π.



To find the distance around the shape, you need to add up the two arc lengths and OQ.

Length of semicircular arc $PQ = \frac{1}{2} \cdot 2\pi r$

$$\approx \frac{1}{\sqrt{2}} \cdot 2^{1} \cdot \frac{22}{\sqrt{7}} \cdot 14^{2}$$
$$= 1 \cdot 22 \cdot 2$$
$$= 44 \text{ cm}$$

Write formula. Substitute. Divide by the common factors, 2 and 7. Simplify. Multiply.

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Length of semicircular arc $PO = \frac{1}{2} \cdot \pi d$

$$\approx \frac{1}{12} \cdot \frac{22}{17}^{11} \cdot 14^2$$
$$= 1 \cdot 11 \cdot 2$$
$$= 22 \text{ cm}$$

Write formula. Substitute. Divide by the common factors, 2 and 7. Simplify. Multiply.

Distance around the shape

- = length of semicircular arc PQ + length of semicircular arc PO + OQ
- = 44 + 22 + 14
- = 80 cm

The distance around the shape is approximately 80 centimeters.

c) The shape of a table top is made up of a semicircle and a quadrant. Find the distance around the table top. Use 3.14 as an approximation for π .



The distance around the table top is approximately 248.4 inches.

Guided Practice

Complete. Use 3.14 as an approximation for π .

1 The circumference of the moon is the approximate distance around a circle with radius 1,736 kilometers. Find the circumference of the moon.

a) Round your answer to the nearest 10 kilometers.

Circumference of moon = $2\pi r$

$$\approx \underline{?} \cdot \underline{?} \cdot \underline{?}$$
$$= \underline{?} \text{ km}$$

The circumference of the moon to the nearest 10 kilometers is _?____ kilometers.

b) Round your answer to the nearest 1,000 kilometers.

The circumference of the moon to the nearest 1,000 kilometers is <u>?</u> kilometers.

2 A greeting card is made up of three semicircles. O is the center of the large semicircle. Sarah wants to decorate the distance around the card with a ribbon. How much ribbon does Sarah need? Round your answer to the nearest inch.

Length of semicircular arc $AB = \frac{1}{2} \cdot 2\pi r$

$$\approx \underline{?} \cdot \underline{?} \cdot \underline{?} \cdot \underline{?} \cdot \underline{?}$$
$$= 1 \cdot \underline{?} \cdot \underline{?}$$
$$= \underline{?} \text{ in.}$$

Semicircular arcs AO and OB have the same length.

Total length of semicircular arcs AO and OB

$$= 2 \cdot \frac{1}{2} \cdot \pi d$$

$$\approx \underline{?} \cdot \underline{?} \cdot \underline{?} \cdot \underline{?}$$

$$= 1 \cdot \underline{?} \cdot \underline{?}$$

Distance around the card

- = length of semicircular arc AB + total length of semicircular arcs AO and OB
- = <u>?</u> + <u>?</u>
- = <u>?</u> in.
- ≈ ? in.

Sarah needs approximately <u>?</u> inches of ribbon.



3 As part of her artwork, Sally bends a length of wire into the shape shown. The shape is made up of a semicircle and a quadrant. Find the length of the wire.

Length of semicircular arc PQ



Distance around the shape

= length of semicircular arc PQ + length of arc RO + RP + OQ

$$= \frac{?}{?} + \frac{?}{?} + \frac{?}{?} + \frac{?}{?}$$
$$= \frac{?}{?} \text{ cm}$$

The length of the wire is approximately <u>?</u> centimeters.

garn Use the formula for area of a circle to solve real-world problems.

a) A jewelry designer is making a pendant. The pendant will be a circular disc (center *O*) with a circular hole cut out of it, as shown. The radius of the disc is 35 millimeters. Find the area of the pendant. Use $\frac{22}{7}$ as an approximation for π .



Area of hole = πr^2 $\approx \frac{22}{7} \cdot 17.5 \cdot 17.5$ = 962.5 mm²

Area of pendant = area of disc - area of hole = 3,850 - 962.5 = 2,887.5 mm²

The area of the pendant is approximately 2,887.5 square millimeters.

A graphic designer creates a design for a company logo. The design is a green semicircle with a white quadrant, as shown. Find the area of the green part of the design. Use 3.14 as an approximation for π .

$$30 \text{ mm} \underbrace{160 \text{ mm}}_{160 \text{ mm}} 160 \text{ mm}}$$
Area of semicircle = $\frac{1}{2} \cdot \pi r^2$
 $\approx \frac{1}{2} \cdot 3.14 \cdot 160^2$
= $\frac{1}{12} \cdot 3.14 \cdot 160 \cdot 160^{80}$
= $1 \cdot 3.14 \cdot 160 \cdot 80$
= $40,192 \text{ mm}^2$
Area of quadrant = $\frac{1}{4} \cdot \pi r^2$
 $\approx \frac{1}{4} \cdot 3.14 \cdot 30^2$
= $\frac{1}{14} \cdot 3.14 \cdot 30^{15} \cdot 30^{15}$
= $1 \cdot 3.14 \cdot 15 \cdot 15$
= 706.5 mm^2

Area of green part

b)

= area of semicircle - area of quadrant

= 39,485.5 mm²

The area of the green part of the design is approximately 39,485.5 square millimeters.

Guided Practice

Complete. Use $\frac{22}{7}$ as an approximation for π .

4 Judy baked a pizza and had part of it for lunch. After the meal, the shape of the remaining pizza is made up of a semicircle and a quadrant. Find the area of the remaining pizza.

Area of quadrant =
$$\frac{1}{4} \cdot \pi r^2$$

 $\approx \frac{1}{4} \cdot \frac{?}{2} \cdot \frac{?}{2}^2$
 $= \frac{1}{4} \cdot \frac{?}{2} \cdot \frac{?}{2} \cdot \frac{?}{2}$
 $= \frac{?}{2} \text{ cm}^2$

Area of semicircle = $\frac{1}{2} \cdot \pi r^2$

$$\approx \frac{1}{2} \cdot \frac{?}{2} \cdot \frac{?}{2} \cdot \frac{?}{2}$$
$$= \frac{1}{2} \cdot \frac{?}{2} \cdot \frac{?}{2} \cdot \frac{?}{2} \cdot \frac{?}{2} \cdot \frac{?}{2}$$
$$= \frac{?}{2} \operatorname{cm}^{2}$$

Area of remaining pizza = area of quadrant + area of semicircle

$$=$$
 ? + ?
 $=$? cm²

The area of the remaining pizza is approximately _____ square centimeters.

5 A rug is made up of a quadrant and two semicircles. Find the area of the rug.

Area of quadrant
$$= \frac{1}{4} \cdot \pi r^2$$

 $\approx \frac{1}{4} \cdot \frac{?}{2} \cdot \frac{?}{2}^2$
 $= \frac{1}{4} \cdot \frac{?}{2} \cdot \frac{?}{2} \cdot \frac{?}{2}$
 $= \frac{?}{10.2}$ in.²

Radius of semicircle = diameter \div 2



42 in.

Total area of two semicircles = $2 \cdot \frac{1}{2} \cdot \pi r^2$

$$\approx \underline{?} \cdot \underline{?} \cdot \underline{?} \cdot \underline{?}^{2}$$
$$= 1 \cdot \underline{?} \cdot \underline{?} \cdot \underline{?}^{2}$$
$$= \underline{?} \text{ in.}^{2}$$

Area of figure = area of quadrant + total area of two semicircles

$$=$$
 ? + ?
 $=$? in.²

The area of the rug is approximately <u>?</u> square inches.

garn Solve real-world problems involving rates and circles.

1 The tire of a car has a radius of 10.5 inches. How many revolutions does the tire need to make for the car to travel 13,200 inches? Use $\frac{22}{7}$ as an approximation for π .





Circumference of tire = $2\pi r$

$$\approx 2 \cdot \frac{22}{7} \cdot 10.5$$
$$= \frac{22}{17} \cdot 21^{3}$$
$$= 66 \text{ in.}$$

The car travels approximately 66 inches with one revolution of the tire.

Number of revolutions = distance \div circumference of tire = 13,200 \div 66 = 200

The tire needs to make approximately 200 revolutions to travel 13,200 inches.

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A field is shaped like the diagram below. It is a rectangle with semicircles at the two ends. There is a running track around the field. Use 3.14 as an approximation for π .



a) Find the length of the running track. Round your answer to the nearest ten meters.

Semicircular arcs AB and DC have the same length.

Total length of semicircular arcs AB and $DC = 2 \cdot \frac{1}{2} \cdot \pi d$ = $1 \cdot \pi d$ $\approx 1 \cdot 3.14 \cdot 64$ = 200.96 m

Length of running track = total length of semicircular arcs AB and DC + AD + BC= 200.96 + 100 + 100 = 400.96 m \approx 400 m Round to the nearest ten meters.

The length of the running track is approximately 400 meters.

b) An athlete ran around the track one time at an average speed of 8 meters per second. How many seconds did it take him to run around the track?

Time taken = distance \div speed $\approx 400 \div 8$ Substitute. = 50 s Divide.

It took the athlete approximately 50 seconds to run one time around the track.

c) A gardener is hired to cut the grass in the field. She cuts the grass at an average rate of 40 square meters per minute. How many hours will she take to finish the entire field? Round your answer to the nearest hour.

The areas of the two semicircles are equal. Radius of circle = diameter $\div 2$ $= 64 \div 2$ = 32 m Total area of two semicircles = $2 \cdot \frac{1}{2} \cdot \pi r^2$ Write formula. Substitute. Divide by $\approx {}^{1}\mathbf{2} \cdot \frac{1}{\mathbf{12}} \cdot 3.14 \cdot 32^{2}$ the common factor, 2. $= 1 \cdot 3.14 \cdot 32 \cdot 32$ Simplify. $= 3.215.36 \text{ m}^2$ Multiply. Area of rectangle $ABCD = \ell w$ $= 6,400 \text{ m}^2$ Substitute. Multiply. Area of field = area of rectangle ABCD + total area of two semicircles = 6,400 + 3,215.36= 9,615.36 m² Time taken by gardener = area of field \div rate of cutting grass ≈ 9.615.36 ÷ 40 Substitute. = 240.384 min Divide by 60 to convert to hours. = 4.0064 h ≈4h Round to the nearest hour. The gardener will take approximately 4 hours to finish the entire field.

Guided Practice

Complete. Use $\frac{22}{7}$ as an approximation for π .

6 The diameter of a bicycle wheel is 60 centimeters. How far does the wheel travel when it makes 35 revolutions? Give your answer in meters.

Circumference of wheel $= \pi d$

Distance traveled = circumference of wheel \cdot number of revolutions

$$= \underline{?} \cdot \underline{?}$$

$$= \underline{?} \text{ cm}$$

$$= \underline{?} \text{ m}$$
Divide by 100 to convert to meters.

The wheel travels approximately <u>?</u> meters.



A park is shaped like the diagram below. It is a rectangle with semicircles at the two ends. There is a running track around the park.



a) The total length of the track is 220 yards. Find the length of \overline{PS} .

The track is made up of semicircular arcs PQ and SR, and sides PS and QR. Semicircular arcs PQ and SR are equal.

Total length of semicircular arcs PQ and SR

$$= 2 \cdot \frac{1}{2} \cdot \pi d$$
$$\approx 1 \cdot \underline{?} \cdot \underline{?}$$

= <u>?</u> yd

The length of \overline{PS} and \overline{QR} are equal.

Total length of track = 220

PS + QR + total length of semicircular arcs PQ and SR = 220

$$PS + QR + \underline{?} = 220$$

$$PS + QR = 220$$

$$PS + QR = 220$$

$$2 \cdot PS = \underline{?}$$

$$PS =$$

The length of \overline{PS} is approximately ____ yards.

b) A jogger runs once around the track in 125 seconds. What is his average speed in yards per second?

The jogger runs _?___yards in 125 seconds.

125 seconds
$$\longrightarrow ?$$

1 second $\longrightarrow ? ? ?$
 $= ? yd$

The average speed of the jogger is <u>?</u> yards per second.

c) A gardener is hired to water the grass in the park. Using a machine, he waters 4 square yards per second. How many minutes will he take to water the entire park? Round your answer to the nearest minute.

Radius = diameter \div 2

The areas of the two semicircles are equal.

Total area of two semicircles = $2 \cdot \frac{1}{2} \cdot \pi r^2$ $\approx 2 \cdot \frac{1}{2} \cdot \frac{?}{2} \cdot \frac{?}{2} \cdot \frac{?}{2}^2$ = $1 \cdot \frac{?}{2} \cdot \frac{?}{2} \cdot \frac{?}{2} \cdot \frac{?}{2}^2$ = $\frac{?}{2} y d^2$

Area of rectangle $PQRS = \ell w$

$$= \underline{?} \cdot \underline{?}$$
$$= \underline{?} yd^{2}$$

Area of park = area of rectangle PQRS + total area of two semicircles

 $= \underline{?} + \underline{?}$ $= \underline{?} yd^{2}$

Time taken = area of park ÷ rate of watering park

 $= \frac{?}{\cdot} \div \frac{?}{\cdot}$ $= \frac{?}{\cdot} s$ $= \frac{?}{\cdot} \min$ $\approx \frac{?}{\cdot} \min$

Round to the nearest minute.

The gardener will take approximately <u>?</u> minutes to water the entire park.

To find the time taken to water the park, think: Rate of watering = area of park ÷ time taken You can write this equation as: Time taken = area of park ÷ rate of watering park Remember to express the answer in minutes, not seconds.



Solve. Show your work.

- 1 The radius of a circular pond is 8 meters. Find its area and circumference. Use 3.14 as an approximation for π .
- 2 The diameter of a metal disc is 26 centimeters. Find its area and circumference. Use 3.14 as an approximation for π.
- **3** The shape of a carpet is a semicircle. Use $\frac{22}{7}$ as an approximation for π .
 - a) Find its area.
 - b) Janice wants to put a fringed border on all sides of the carpet. How many feet of fringe are needed?





4 The circumference of the rim of a wheel is 301.44 centimeters. Find the diameter of the rim. Use 3.14 as an approximation for π.

A Japanese fan is made out of wood and cloth. The shape of the fan is made up of two overlapping quadrants. What is the area of the portion that is made

of cloth? Use $\frac{22}{7}$ as an approximation for π .



6 A pancake restaurant serves small silver-dollar pancakes and regular-size pancakes. Use 3.14 as an approximation for π .



- a) What is the area of a small silver dollar-pancake? Round your answer to the nearest tenth of an inch.
- **b**) What is the area of a regular-size pancake? Round your answer to the nearest tenth of a square inch.
- c) If the total price of 6 small silver-dollar pancakes is the same as the total price of 3 regular-size pancakes, which is a better deal?

7 A park is shaped like a rectangle with a semicircle on one end, and another semicircle cut out of one side.

- a) Find the distance around the park.
- b) Find the area of the park. Use $\frac{22}{7}$ as an approximation for π .





- Find the area of the sidewalk. a)
- b) 0.8 bag of concrete will be needed for every square foot of the new sidewalk. What is the minimum number of bags needed?





🔢 🧿 The diagram shows an athletic field with a track around it. The track is 4 feet wide. The field is a rectangle with semicircles at the two ends. Find the area of the track. Use 3.14 as an approximation for π .



10 The petal of a paper flower is created by cutting along the outlines of two overlapping quadrants within a square. Use 3.14 as an approximation for π .

- Find the distance around the shaded part. a)
- Find the area of the shaded part. b)



11 Wheels A and B are placed side by side on a straight road. The diameter of wheel A is 56 inches. The diameter of wheel B is 35 inches. Suppose each wheel makes 15 revolutions. Find the distance between the wheels after they have made these 15 revolutions.



12 Nine identical circles are cut from a square sheet of paper whose sides are 36 centimeters long. If the circles are as large as possible, what is the area of the paper that is left after all the circles are cut out? Use 3.14 as an approximation for π.

13 A designer drew an icon as shown below. *O* is the center of the circle, and \overline{AB} is a diameter. Two semicircles are drawn in the circle. If *AB* is 28 millimeters, find the area of the shaded part. Use $\frac{22}{7}$ as an approximation for π .



Use graph paper. Solve.

- 14 Mary wants to draw the plan of a circular park on graph paper. The coordinates of the center of the park are A (3, 4). The circle has a radius of 3 units.
 - a) Use a compass and draw the plan of the circular park on graph paper.
 - **b**) Assume that the *y*-axis points north and south. A barbecue pit is located at the northernmost part of the park. Plot and label the location of the barbecue pit as point *B*. Give the coordinates of point *B*.
 - c) Connect points *A*, *B*, and the origin to form a triangle. Find the area of the triangle.

A wire is bent to make the shape below. The shape is made up of four identical circles. Each circle intersects two other circles. The four circles meet at a common point *T*, which is the center of square *PQRS*. Use ²²/₇ as an approximation for π.

- a) Find the length of the wire.
- **b)** Find the area of the whole shape.





2 A cushion cover design is created from a circle of radius 7 inches, and 4 quadrants. Find the total area of the shaded parts of the design.

Use $\frac{22}{7}$ as an approximation for π .



Two identical wheels are placed along a straight path so that their centers are 9.31 meters apart. The radius of each wheel is 3.5 centimeters. They are pushed towards each other at the same time, each making one revolution per second. How long does it take for them to knock into each other? Use $\frac{22}{7}$ as an approximation for π .



Chapter Wrap Up

Concept Map



Key Concepts

- All radii of a circle are equal.
- A diameter of a circle is twice its radius.
- The number π is the ratio of the circumference to the diameter of a circle.

Chapter Review/Test

Concepts and Skills



Find the distance around each semicircle. Use $\frac{22}{7}$ as an approximation for π .



Find the distance around each quadrant. Round your answer to the nearest tenth. Use 3.14 as an approximation for π .





Solve. Show your work.

- 7 The diameter of a flying disc is 10 inches. Find the circumference and area of the disc. Use 3.14 as an approximation for π .
- 8 The area of a compact disc is $452\frac{4}{7}$ square centimeters. What is the diameter of the compact disc? Use $\frac{22}{7}$ as an approximation for π .
- 9 The circumference of a circular table is 816.4 centimeters. Find the radius of the table. Use 3.14 as an approximation for π .

Problem Solving

Solve. Show your work.

A water fountain shoots up a jet of water. The water falls back down onto the ground in the shape of a circle. Michelle wants the circle of water on the ground to be 0.7 meter wider on each side. She gradually increases the strength of the water jet. The area of the circle of water increases at

0.2 square meter per second. Use $\frac{22}{7}$ as an approximation for π .

- a) Find the area of the original circle of water.
- **b)** Find the area of the larger circle of water.
- c) How long does it take for the original circle of water to become the larger circle of water? Round your answer to the nearest second.



- 11 A machine in an assembly line stamps pieces of metal. The stamping plate on the machine travels in a path shaped like the arc of a quadrant as the stamping plate opens and closes. It takes the machine 5 seconds to open and close the stamping plate one time. Use $\frac{22}{7}$ as an approximation for π .
 - a) Find the total distance the outside edge of the stamping plate travels when the machine opens and closes one time.
 - **b**) Find the speed of the stamping plate's outside edge in centimeters per second.
 - c) Assume the machine starts and ends in an open position. How many seconds will it take the machine to stamp 500 pieces of metal?



12 The figure shows four identical quadrants enclosed in a square. The side length of the square is 20 inches. Find the area of the blue part. Use 3.14 as an approximation for π .



13 The figure shows 3 identical circles. X, Y, and Z are the centers of the circles, and the radius of each circle is 15 feet. $\frac{1}{6}$ of each circle is shaded. What is the total area of the shaded portion? Round your answer to the nearest tenth of a foot. Use 3.14 as an approximation for π .



14 The figure is made up of one semicircle and two quadrants. The distance around the figure is 97.29 inches. Find the value of k. Use 3.14 as an approximation for π .

