

# 12.4

## Real-World Problems: Surface Area and Volume

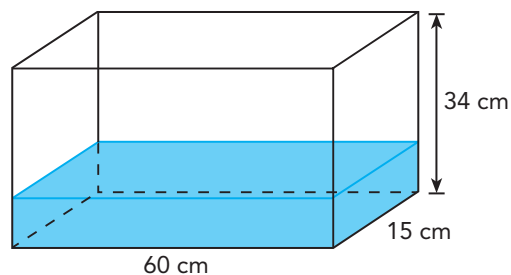
### Lesson Objective

- Solve problems involving surface area and volume of prisms.

Learn

### Solve word problems about the volume of rectangular prisms.

A rectangular fish tank 60 centimeters by 15 centimeters by 34 centimeters is  $\frac{1}{3}$  full of water. Find the volume of water needed to fill the tank completely.



Volume of water needed  
= Volume of empty space in the tank

$$\begin{aligned} \text{Height of empty space} &= \frac{2}{3} \cdot 34 \\ &= \frac{68}{3} \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Volume of water needed to fill the tank} &= 60 \cdot 15 \cdot \frac{68}{3} \\ &= 60 \cdot 5 \cdot 68 \\ &= 20,400 \text{ cm}^3 \end{aligned}$$

Write equation.

Divide out common factor 3.

Multiply.

To fill the tank, 20,400 cubic centimeters more water are needed.

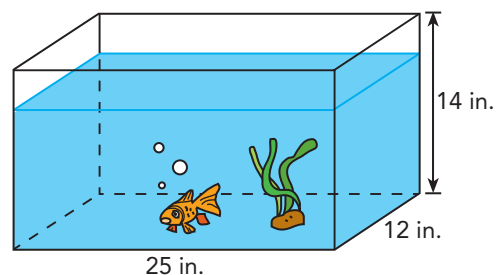
### Guided Practice

Complete.

- Find the volume of water needed to fill three fourths of the aquarium.

$$\begin{aligned} \text{Height of water needed} &= \frac{3}{4} \cdot \underline{\quad? \quad} \\ &= \underline{\quad? \quad} \text{ in.} \end{aligned}$$

$$\begin{aligned} \text{Volume of water needed} &= \underline{\quad? \quad} \cdot \underline{\quad? \quad} \cdot \underline{\quad? \quad} \\ &= \underline{\quad? \quad} \text{ in.}^3 \end{aligned}$$

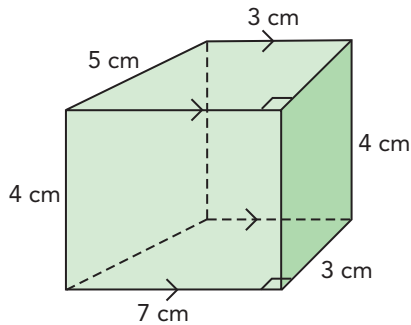


The aquarium needs to have  $\underline{\quad? \quad}$  cubic inches of water added to it to be  $\frac{3}{4}$  full.

## Solve word problems about surface area and volume of non-rectangular prisms.

A block of wood is a prism and has the dimensions shown in the diagram below.

- a) Find the volume of the block of wood.



The base of the prism is a trapezoid.

Area of base

$$\begin{aligned}
 &= \frac{1}{2} h(\text{sum of lengths of parallel sides}) \\
 &= \frac{1}{2} \cdot 3 \cdot (3 + 7) \\
 &= \frac{1}{2} \cdot 3 \cdot 10 \\
 &= 15 \text{ cm}^2
 \end{aligned}$$

Volume

$$\begin{aligned}
 &= Bh \\
 &= 15 \cdot 4 \\
 &= 60 \text{ cm}^3
 \end{aligned}$$

Because all prisms have uniform cross sections when sliced parallel to the bases, you can use the formula  $V = Bh$ .



The volume of the block of wood is 60 cubic centimeters.

- b) Find the surface area of the wooden block.

Surface area of wooden block

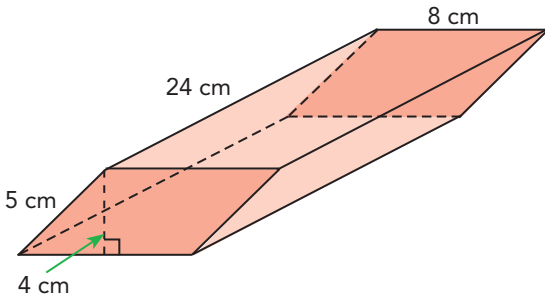
$$\begin{aligned}
 &= \text{perimeter of base} \cdot \text{height} + \text{total area of two bases} \\
 &= (7 + 3 + 3 + 5) \cdot 4 + (2 \cdot 15) \\
 &= 18 \cdot 4 + 30 \\
 &= 72 + 30 \\
 &= 102 \text{ cm}^2
 \end{aligned}$$

The surface area of the wooden block is 102 square centimeters.

## Guided Practice

Complete.

- 2 A metal bar has bases that are parallelograms.



- a) Find the volume of the metal bar.

$$\begin{aligned}\text{Area of parallelogram} &= \text{base of parallelogram} \cdot \text{height of parallelogram} \\ &= \underline{\quad} \cdot \underline{\quad} \\ &= \underline{\quad} \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Volume of metal bar} &= \text{base of prism} \cdot \text{height of prism} \\ &= \underline{\quad} \cdot \underline{\quad} \\ &= \underline{\quad} \text{ cm}^3\end{aligned}$$

The volume of the metal bar is  $\underline{\quad}$  cubic centimeters.

- b) Find the surface area of the metal bar.

$$\begin{aligned}\text{Surface area of metal bar} &= \text{perimeter of base} \cdot \text{height} + \text{total area of 2 bases} \\ &= (\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}) \cdot \underline{\quad} + \underline{\quad} \cdot \underline{\quad} \\ &= \underline{\quad} \cdot \underline{\quad} + \underline{\quad} \\ &= \underline{\quad} + \underline{\quad} \\ &= \underline{\quad} \text{ cm}^2\end{aligned}$$

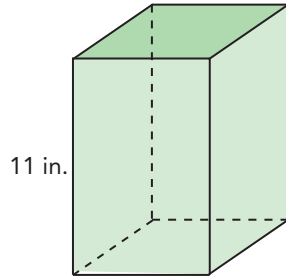
The surface area of the metal bar is  $\underline{\quad}$  square centimeters.

## Learn

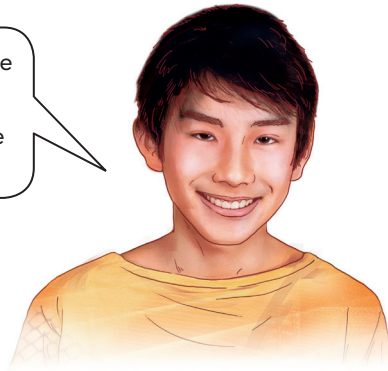
**Solve word problems about prisms with missing dimensions.**

A square prism of height 11 inches has a volume of 539 cubic inches.

- a) Find the length of each side of the square base.



To find the length of the square base, you can first find the area of the square base.



$$V = Bh$$

Write formula.

$$539 = B \cdot 11$$

Substitute.

$$\frac{539}{11} = B \cdot \frac{11}{11}$$

Divide each side by 11.

$$49 = B$$

Simplify.

The area of the square base is 49 square inches.

$$\begin{aligned} \text{Length of each side of base} &= \sqrt{49} \\ &= 7 \text{ in.} \end{aligned}$$

The length of each side of the square base is 7 inches.

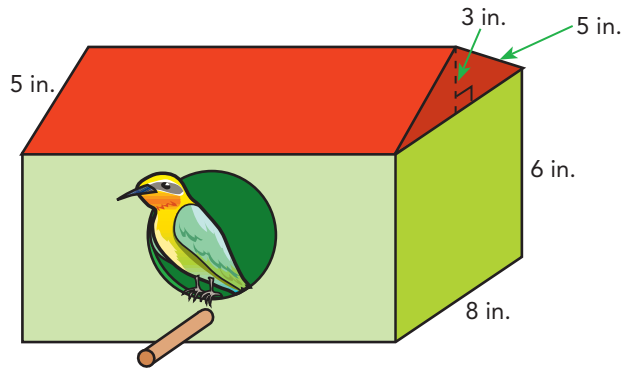
- b) Find the surface area of the prism.

$$\begin{aligned} \text{Surface area of prism} &= \text{perimeter of base} \cdot \text{height} + \text{area of two bases} \\ &= (7 + 7 + 7 + 7) \cdot 11 + 2 \cdot 49 \\ &= 28 \cdot 11 + 98 \\ &= 308 + 98 \\ &= 406 \text{ in.}^2 \end{aligned}$$

The surface area of the prism is 406 square inches.

## Solve word problems about non-rectangular prisms with missing dimensions.

Jacob is making a wooden birdhouse. The birdhouse is a prism with bases that are pentagons, and has the dimensions shown in the diagram. The volume of the prism is 720 cubic inches.



- a) Find the height of the prism.

$$\begin{aligned}
 &\text{Area of pentagonal base} \\
 &= \text{area of triangle} + \text{area of rectangle} \\
 &= \left(\frac{1}{2} \cdot 8 \cdot 3\right) + 8 \cdot 6 \\
 &= 12 + 48 \\
 &= 60 \text{ in.}^2
 \end{aligned}$$

$$V = Bh$$

Write formula.

$$720 = 60 \cdot h$$

Substitute.

$$\frac{720}{60} = h \cdot \frac{60}{60}$$

Divide each side by 60.

$$12 = h$$

Simplify.

The height of the prism is 12 inches.

- b) Find the surface area of the prism.

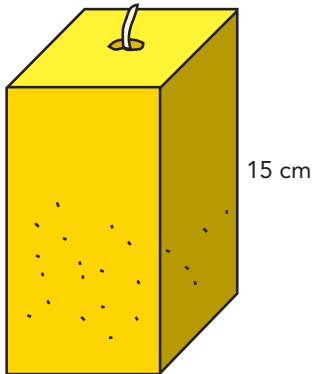
$$\begin{aligned}
 &\text{Surface area of the prism} \\
 &= \text{perimeter of base} \cdot \text{height} + \text{area of two bases} \\
 &= (5 + 6 + 8 + 6 + 5) \cdot 12 + 2 \cdot 60 \\
 &= 30 \cdot 12 + 120 \\
 &= 360 + 120 \\
 &= 480 \text{ in.}^2
 \end{aligned}$$

The surface area of the prism is 480 square inches.

## Guided Practice

Complete.

- 3 A candle is a square prism. The candle is 15 centimeters high, and its volume is 960 cubic centimeters.



- a) Find the length of each side of the square base.

$$V = Bh$$

$$\underline{\quad ? \quad} = B \cdot \underline{\quad ? \quad}$$

$$\underline{\quad ? \quad} \div \underline{\quad ? \quad} = \underline{\quad ? \quad} \cdot B \div \underline{\quad ? \quad}$$

$$\underline{\quad ? \quad} = B$$

Length of each side of base

$$= \underline{\quad ? \quad}$$

$$= \underline{\quad ? \quad} \text{ cm}$$

The length of each side of the square base is  $\underline{\quad ? \quad}$  centimeters.

- b) Find the surface area of the candle.

Surface area of candle

= perimeter of base  $\cdot$  height + area of two bases

$$= (\underline{\quad ? \quad} + \underline{\quad ? \quad} + \underline{\quad ? \quad} + \underline{\quad ? \quad}) \cdot \underline{\quad ? \quad} + \underline{\quad ? \quad} \cdot \underline{\quad ? \quad}$$

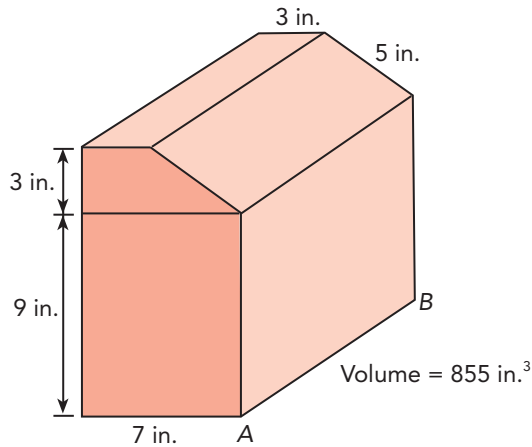
$$= \underline{\quad ? \quad} \cdot \underline{\quad ? \quad} + \underline{\quad ? \quad}$$

$$= \underline{\quad ? \quad} + \underline{\quad ? \quad}$$

$$= \underline{\quad ? \quad} \text{ cm}^2$$

The surface area of the candle is  $\underline{\quad ? \quad}$  square centimeters.

- 4 A storage chest is a prism with bases that are pentagons. The diagram shows some of the dimensions of the storage chest. The volume of the storage chest is 855 cubic inches.



- a) Find the height  $AB$  of the prism. Round your answer to the nearest hundredth.

$$\begin{aligned}
 &\text{Area of pentagonal base} \\
 &= \text{area of trapezoid} + \text{area of rectangle} \\
 &= \frac{1}{2} \cdot \underline{\quad} \cdot (\underline{\quad} + \underline{\quad}) + \underline{\quad} \cdot \underline{\quad} \\
 &= \underline{\quad} + \underline{\quad} \\
 &= \underline{\quad} \text{ in.}^2
 \end{aligned}$$

$$\begin{aligned}
 V &= Bh \\
 \underline{\quad} &= \underline{\quad} \cdot h \\
 \underline{\quad} \div \underline{\quad} &= \underline{\quad} \cdot h \div \underline{\quad} \\
 \underline{\quad} &\approx h
 \end{aligned}$$

The height of the prism is approximately  $\underline{\quad}$  inches.

- b) Find the surface area of the prism. Round your answer to the nearest hundredth.

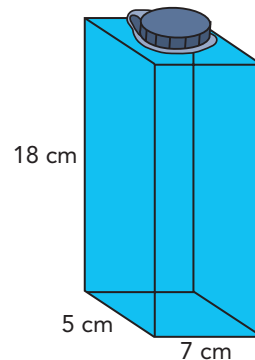
$$\begin{aligned}
 &\text{Surface area of prism} \\
 &= \text{perimeter of base} \cdot \text{height} + \text{area of two bases} \\
 &\approx (\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}) \cdot \underline{\quad} + \underline{\quad} \cdot \underline{\quad} \\
 &= \underline{\quad} \cdot \underline{\quad} + \underline{\quad} \\
 &= \underline{\quad} + \underline{\quad} \\
 &= \underline{\quad} \text{ in.}^2
 \end{aligned}$$

The surface area of the prism is approximately  $\underline{\quad}$  square inches.

## Practice 12.4

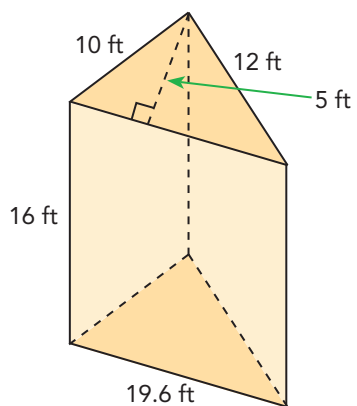
Solve.

- 1 Savannah has a water bottle that is a rectangular prism. The bottle measures 7 centimeters by 5 centimeters by 18 centimeters and she filled it completely with water. Then, she drank  $\frac{1}{4}$  of the volume of water in her water bottle. How many cubic centimeters of water were left in the water bottle?

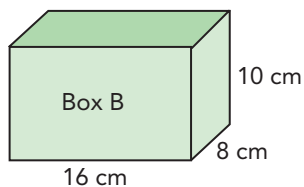
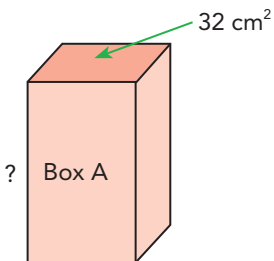


- 2 A rectangular prism has a square base with edges measuring 8 inches each. Its volume is 768 cubic inches.
- Find the height of the prism.
  - Find the surface area of the prism.

- 3 A triangular prism has the measurements shown.
- Find the volume of the prism.
  - Find the surface area of the prism.



- 4 The volume of Box A is  $\frac{2}{5}$  the volume of Box B. What is the height of Box A if it has a base area of 32 square centimeters?





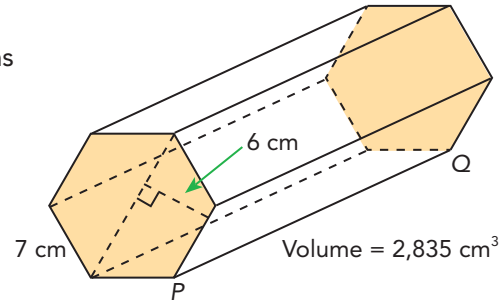


5 The ratio of the length to the width to the height of an open rectangular tank is 10 : 5 : 8. The height of the tank is 18 feet longer than the width.

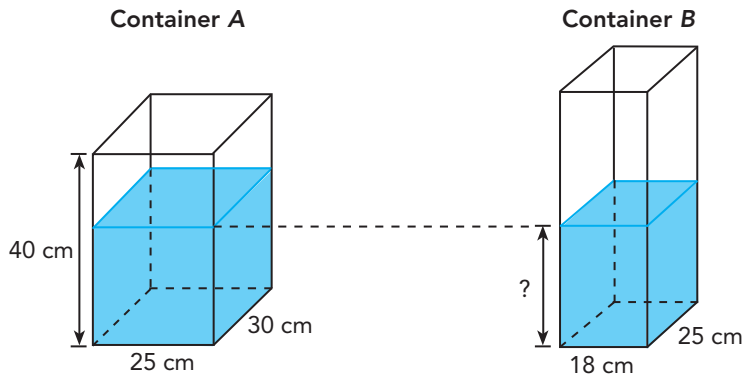
- Find the volume of the tank.
- Find the surface area of the open tank.

6 Janice is making a gift box. The gift box is a prism with bases that are regular hexagons, and has the dimensions shown in the diagram.

- Find the height  $PQ$  of the prism.
- Find the surface area of the prism.



7 Container A was filled with water to the brim. Then, some of the water was poured into an empty Container B until the height of the water in both containers became the same. Find the new height of the water in both containers.

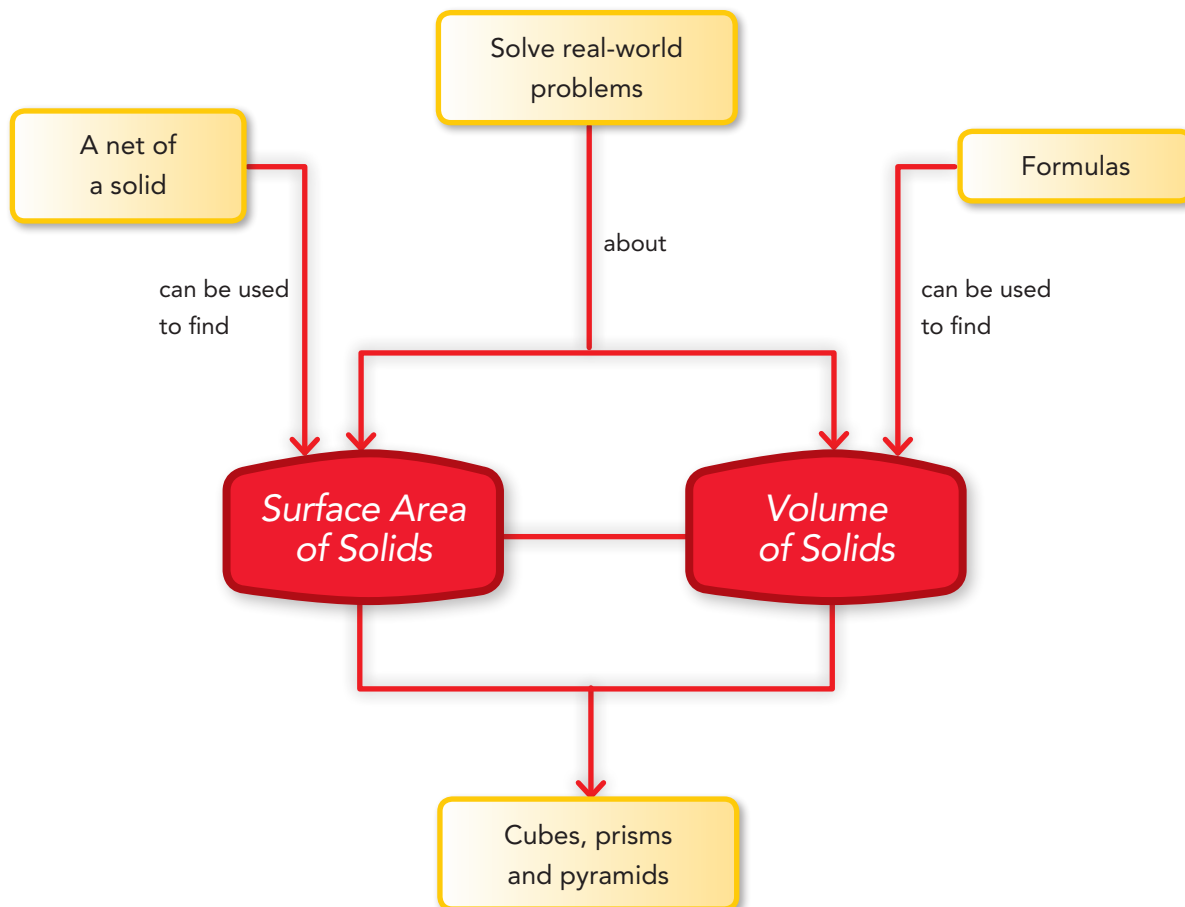


## Brain @ Work

- The volume of a cube is 100 cubic inches. If each of the edges is doubled in length, what will be the volume of the cube?
- The volume of a cube is  $x$  cubic feet and its surface area is  $x$  square feet, where  $x$  represents the same number. Find the length of each edge of the cube.

# Chapter Wrap Up

## Concept Map



## Key Concepts

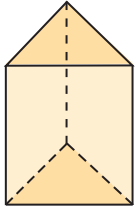
- ▶ The volume of a rectangular prism is the product of its length, width, and height.
- ▶ The volume of any prism is the product of the area of its base and its height.
- ▶ The surface area of a prism or pyramid is the sum of the areas of its faces.

# Chapter Review/Test

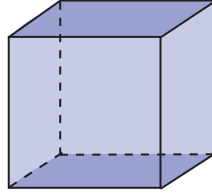
## Concepts and Skills

Match each of the solid figures to its net.

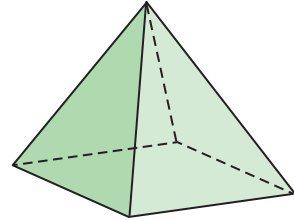
1



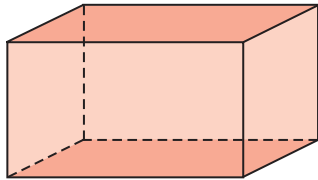
2



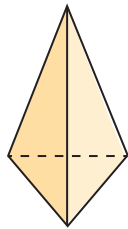
3



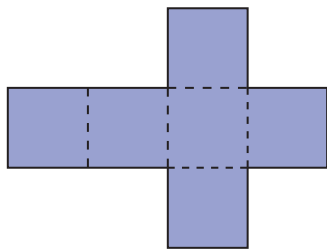
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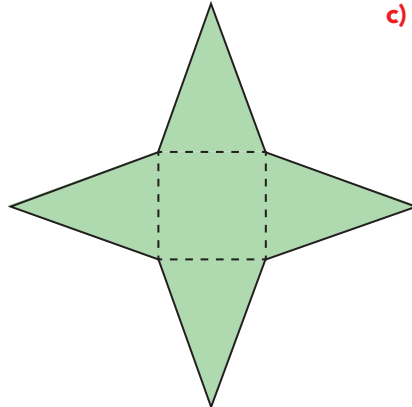
5



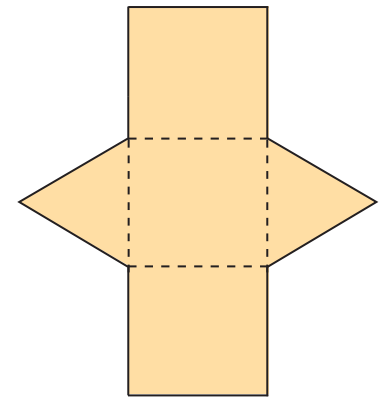
a)



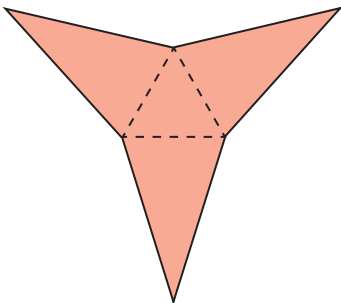
b)



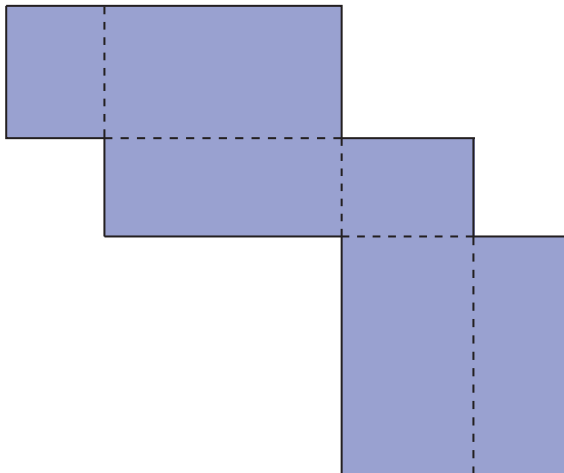
c)



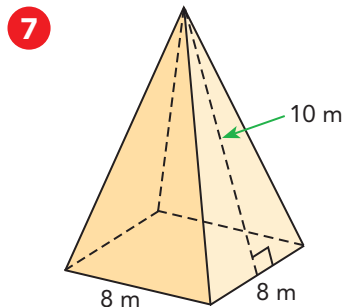
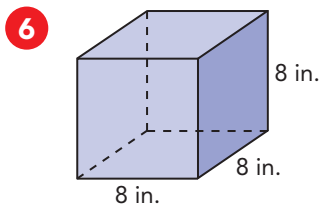
d)



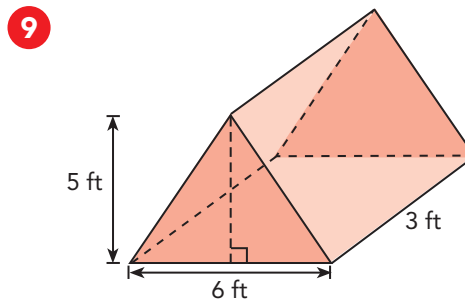
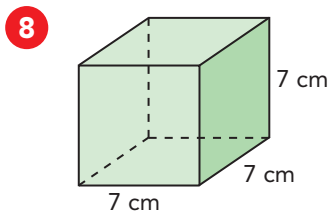
e)



Find the surface area of each solid.



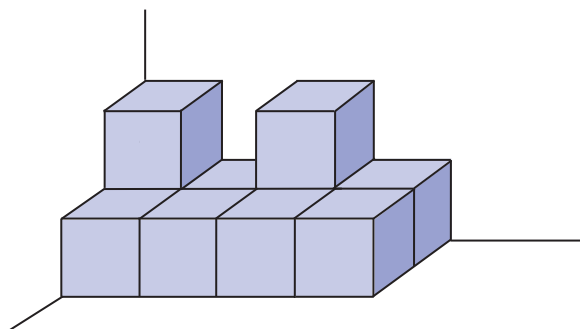
Find the volume of each prism.



Solve.

10 The solid below is made up of cubes, each of which has an edge length of 3 inches.

- a) What is the volume of one cube?
- b) What is the volume of the solid figure?



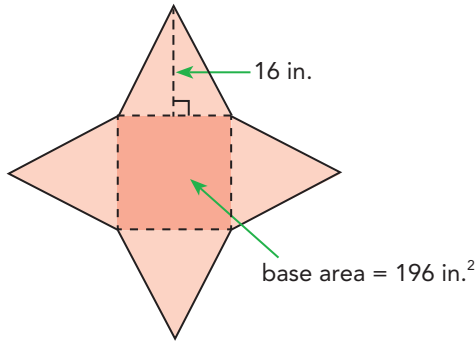
## Problem Solving

Solve.

11 A fish tank is 50 centimeters long, 30 centimeters wide, and 40 centimeters high. It contains water up to a height of 28 centimeters. How many more cubic centimeters of water are needed to fill the tank to a height of 35 centimeters?

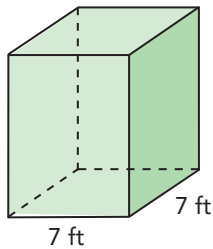


- 12 Find the surface area of a square pyramid given that its base area is 196 square inches and the height of each of its triangular faces is 16 inches.

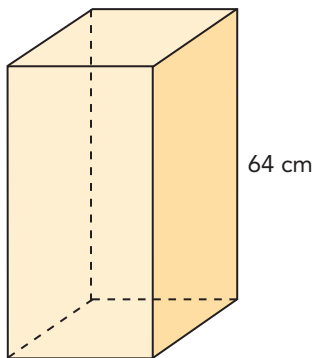


- 13 The volume of a rectangular prism is 441 cubic feet. It has a square base with edges that are 7 feet long.

- a) Find the height of the prism.  
b) Find the surface area of the prism.

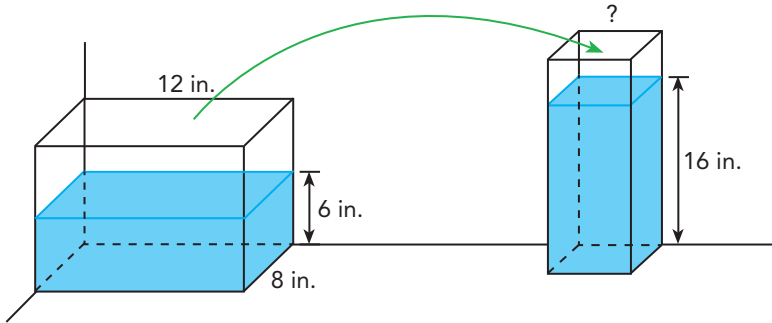


- 14 The volume of a rectangular tank with a square base is 63,908 cubic centimeters. Its height is 64 centimeters. Find the length of an edge of one of the square bases. Round your answer to the nearest tenth of a centimeter.

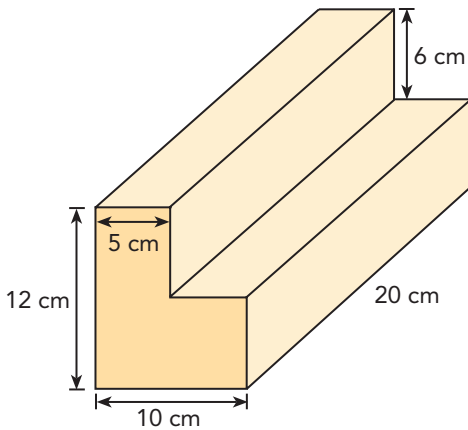




- 15** A rectangular container has a base that is 12 inches long and 8 inches wide. The container is filled with water to a height of 6 inches. If all the water is poured into a second container with a square base, it will rise to a height of 16 inches. What is the length of one edge of the square base of the second container?



- 16** Find the surface area and the volume of the prism.



- 17** Find the surface area and the volume of the prism.

